Name: Yinsheng Dong

Student Number: 11148648

NSID: yid164

Lecture Section: CMPT 260

1. Consider the following expression

∀xP(x) ∧ Q(x) ↔ (∃xR(x) → ∀x((S(x) ∧ Y (y)) ∨ ∃y(U(y) ∨ ∼ T(x))))

1. For each occurrence of each variable, indicate whether the variable is free or bound. If the variable is bound, indicate whether it is bound to a ∀ or to a ∃.

P(x) bounded by ∀

Q(x) is a free variable

R(x) bounded by ∃

S(x) bounded by ∀

Y(y) is a free variable

U(y) is bounded by ∃

~T(x) is bounded by ∀

1. Rename the variables so that distinct names are used for each distinct variable.

∀xP(x) = P

∀xS(x) = S

∀xT(x) = T

∃xR(x) = R

∃xU(y) = U

So, we get that ∀xP ^ Q(x) 🡨🡪(∃xR 🡪(∀xS ^Y(y) V (∃yU V ~∀xT).

1. Show formally that ∼ ∃y(∀x∃zP (x, y, z) ∨ ∃x∀zQ(x, y, z)) is logically equivalent to ∀y(∃x∀z ∼P(x,y,z) ∧∀x∃z ∼Q(x,y,z))

∼ ∃y(∀x∃zP (x, y, z) ∨ ∃x∀zQ(x, y, z)) Premise

∀y ~(∀x∃zP (x, y, z) ∨ ∃x∀zQ(x, y, z)) Negation

∀y(~∀x∃z P(x,y,z) ∧ ~∃x∀z Q(x,y,z)) D.M law

∀y(∃x∀z ∼P(x,y,z) ∧∀x∃z ∼Q(x,y,z)) Negation

1. Find an interpretation to show that the following argument form is not valid. (∀x(P (x) → Q(x)) ∧ (∀x(P (x) → R(x)))) → ∀x((Q(x) → R(x))).

From the variables that questions gave, the argument has a domain ∀x that is unique individuals for each variable, each predicate of the expression defined, and all variables of the expression are bounded by ∀x.

If it is valid, then the expression should be true for all interpretations. So we get a chart.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | P(x) | Q(x) | R(x) | P(x)🡪Q(x) | P(x)🡪R(x) | (P(x)🡪Q(x)) ^(P(x)🡪R(x)) | Q(x)🡪R(x) |
| a | T | T | T | T | T | T | T |
| d | T | T | F | F | F | F | T |

The last column shows that ∀x((Q(x)🡪R(x) is true, but from the 5th column ∀x(P(x)🡪Q(x)) is false, the 6th column ∀x(P(x)🡪R(x)) is false, so the 7th column ∀x ((P(x)🡪Q(x))^(P(x)🡪R(x))) is false. Thus, the ∀x((Q(x) → R(x))) expression is not valid.